The Islamia University Of Bahawalpur,

Department of Computer Science & IT

Bahawalnagar Campus

Course: Numerical Analysis **Program:** BSCS-V (Spring 2020)

Lec-2

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Equation: An equation is a propositional form that expresses the equality, that can be either true or false, b/w the two mathematical expression.

An equation is a statement with equality sign "=".

If two algebraic expressions are linked by equality sign, forms an equation

e.g. (i) $ax+b = cx+d$

$$\frac{dy}{dx} + y\cos x = \sin x$$

Differential Equation:

An equation involving one dependent variable and its derivatives with respect to one or more independent variables, is called a differential equation.

e.g.

(i) $$\frac{dy}{dx} + y\cos x = \sin x$$

(ii) $$\frac{d^2y}{dx^2} + xy\left(\frac{dy}{dx}\right)^2 = 0$$

(iii) $$\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2} = \frac{d^2y}{dx^2}$$

(iv) $$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = nz$$

(v) $$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$

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⇒ Ordinary Differential Equations (O.D

:- A differential equation, in which ordinary derivatives of the dependent variable with respect to a single independent variable occur, is called an ordinary differential equation. e.g equations (i), (ii), (iii) are example of O.D.E's

⇒ Partial Differential Equations (P.D.E

A differential involving partial derivatives of The dependent variable with respect to more than one independent variable is called partial differential equation.

e.g $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu$

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 z}{\partial z^2} = 0$

⇒ Order of Differential Equation :-

is The order of the highest derivative that occurs/appear in the equation.

⇒ Degree Of Different Equation :- :-

The degree of a differential

equation is the degree of the highest order derivative that appears in the equation (Dependent variable and its derivative should be free from radicals & fractions)

e.g. $\frac{dy}{dx} + y\cos x = \sin x$ order=1 degree=1

$\frac{d^2y}{dx^2} + xy\left(\frac{dy}{dx}\right)^2 = 0$ order - 2 degree - 1

$\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2} = \frac{d^2y}{dx^2}$ order - 2 degree - 2

=> Linear Equations:- An equation in which the variable only appears to the first power.

e.g. $11x+3 = 8x+94 \rightarrow S.S=\{7\}$

=> Linear Differential Equation
An ordinary differential equation

$$F\left(x, y, \frac{dy}{dx}, \ldots, \frac{d^ny}{dx^n}\right) = 0$$

is said to be linear if 'F' is a linear function of the variable $x, \frac{dy}{dx}, \frac{d^2y}{dx^2}, \ldots, \frac{d^ny}{dx^n}$

(6) (4)

thus the general linear ordinary differential equation of order n is

$$a_0(x)\frac{d^ny}{dx^n} + a_1(x)\frac{d^{n-1}y}{dx^{n-1}} + \cdots + a_{n-1}(x)\frac{dy}{dx} + a_n(x)y = g(x)$$

$a_0(x)$ is not identically zero.

e.g $\frac{d^3y}{dx^3} + 4\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 3y = \cos x$

i.e ordinary, third order, 1st degree, linear

⇒ Non-Linear Differential Eq:

A differential eq which is not linear is called a non-linear differential equation.

e.g

$$\frac{d^3y}{dx} + 2e^x\frac{d^2y}{dx^2} + y\frac{dy}{dx} = x^3$$

Non Linear Term

⇒ Solution:- The solution of an equation is the value or set of values that a variable can take on such that they make the equation true.

Solution of a Diff. Eq.

A solution of a differential equation is a relation b/w the variables, not containing derivatives, such that this relation and the derivatives obtained from it satisfy the given differential equation.

For exp. $\frac{dy}{dx} = -\lambda y$

has solution $y = Ce^{-\lambda x}$

where 'c' is an arbitrary constan

And (2nd exp)

$$\frac{d^2y}{dx^2} + y = 0$$

has the solutions

$y = A\cos x$, $y = B\sin x$

& $y = A\cos x + B\sin x$

where 'A' & 'B' are arbitrary constants

Note:- Number of constants involves in solution, equals to order of the Differential Eq.

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Solution (Integral)

- General solution
- particular solution

⇒ General Solution:- is a solution which contains the number of arbitrary constants equal to the order of the differential equation is called general solution.

⇒ Particular Solution:- A solution obtained from the general solution by giving particular values to the constants is called particular solution of differential equation.

⇒ Integral curve

The graph of a particular solution (integral) is called integral graph or curve.

Importance of Differential Eq.

Differential equations occur in mathematical formulation of many problems in computer Science & engineering. Some such problems are

(i) Determination of curves which give geometerical properties.
(ii) Study of chemical reactions
(iii) Finding the charge or current in an electric circuits.
(iv) Determining the motion of a projectile rocket, satellite or planet → objects
v) D.Eq(s) helps us for studying motion of every Mass & Motion → 2nd-law (M)e

→ Solution of One Variable equations:

An equation involves only one variable called one variable equation.

e.g. $2x^2 + 2x - 4 = 0$

$ax^2 + bx + c = 0$ Quadratic eq.

$4x - 8 = 4$

⇒ It may linear or non linear

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Solution of non linear eq.

For solution of this type of equation there some common method.

(i) Quadratic Formula
(ii) Factorization
(iii) Completing Square formula.
(iv) Taking common Terms

For exp $2x^2+2x-4=0$ — (i)

(i) Quadratic formula

$$ax^2+bx+c=0 \quad \text{— (ii)}$$

By comparing equ (i) & (ii)

$$a=2 \; ; \; b=2 \; ; \; c=-4$$

Since

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-(2) \pm \sqrt{(2)^2-4(2)(-4)}}{2(2)}$$

$$x = \frac{-2 \pm \sqrt{4+32}}{4}$$

$$x = \frac{-2 \pm \sqrt{36}}{4}$$

$$x = \frac{-2 \pm 6}{4}$$

$x = \frac{-2+6}{4}$; $x = \frac{-2-6}{4}$

$x = 4/4$; $x = -\frac{8}{4}$

$x = 1$; $x = -2$ S.S = {1, -2}

(ii) Factorization Method

$$2x^2+2x-4=0$$
$$2x^2+4x-2x-4=0$$
$$2x(x+2)-2(x+2)=0$$
$$(x+2)(2x-2)=0$$
$$x+2=0 \;;\; 2x-2=0$$
$$\boxed{x=-2} \;;\; 2x=2$$
$$\Rightarrow x=\frac{2}{2}$$
$$\boxed{x=1}$$

$p = -8x^2$

$2 \mid 8$

$2 \mid 4$

$2 \mid 2$

$2, 2, 2$

$+4, -2$

$$S.S = \{1, -2\}$$

(iii) Completing Square method.

For exp. $9x^2-30x+25=0$
$$(3x)^2-2\times(3x)(5)+(5)^2=0$$
$$(3x-5)^2=0$$
$$(3x-5)(3x-5)=0$$
$$3x-5=0 \;;\; 3x-5=0$$
$$3x=5$$
$$\Rightarrow x=5/3$$
$$S.S=\{5/3\}$$

(iv) Taking Common ~~Term~~

For exp. $x^2+2x=0$
$$x(x+2)=0$$

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$\boxed{x=0}$; $x+9=0$

$\boxed{x=-9}$

S.S $= \{0, -9\}$

These solutions must satisfy the give equations..

Solutions of Linear equation in one variable

Finding the values that a variable can be satisfy a given equation is a delicate process.

def: The equation in one variable of degree one is called a "Linear eq. in one variable"

eg $4(x-5)+9x = 3x+7$

$5x-2 = 3x+5$

1st Method:-

To maintain equality we can

(i) Add or subtract the same value on both sides of the eq.

(ii) Multiply or divide both sides of eq. by same value

For exp.

$$5x-2 = 3x+5$$

$$5x-2+2 = 3x+5+2$$

Add on b

$5x = 3x + 7$

$5x - 3x = 3x - 3x + 7$ Substract $3x$ on b.s

$2x = 7$

$\frac{2x}{2} = \frac{7}{2}$ Divide by '2'

$\boxed{x = 7/2}$

2nd Method : In case of all single variable linear equations

To solve these eq.

we carry out the following step

(i) Carry out all distributions.

(ii) Regroup the variables on one side of the equal sign and the number on other.

(iii) Divide both sides by the coefficient of the variable (Along these steps do possible simplificat

For Exp.

$$4(x-5) + 2x = 3x + 7$$

→ Carry out distributions

$$4x - 20 + 2x = 3x + 7$$

$$6x - 20 = 3x + 7$$ (Simplificatio

Regrouping $6x - 20 - 3x = 3x + 7 - 3x$

Substrac $3x$ on both s

$$3x - 20 = 7$$

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By Adding 20 on b.s

$3x - 20 + 20 = 7 + 20$

$3x = 27$ By simplification

→ Divide the two sides by the coefficient of the variable

$$\frac{3x}{3} = \frac{27}{3}$$

$x = 9$ by simplification

S.S = $\{9\}$

Exp 2 − $6(2x+3) + x - 7 = 3(5x+7) + 2$

Sole by above method

Exp. 3 $5y - 4 = -2(2y+2)$

Best of Luck